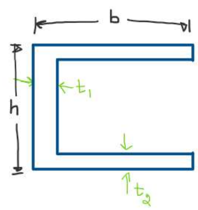
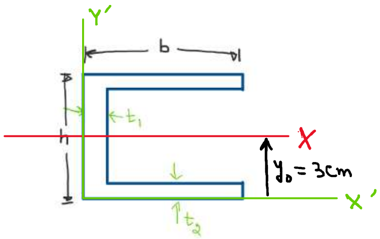
Solution of Moment of Inertia of a C section

Find the moment of Inertia I_x and I_y. x and y axis pass through the centroid(center of gravity).

$b = 10\,cm;\ h = 6\,cm;\ t_1 = 2\,cm;\ t_2 = 1\,cm$



Step-1: Find the center of gravity of hollow C section:



$b = 10\,cm;\ h = 6\,cm;\ t_1 = 2\,cm;\ t_2 = 1\,cm$

Symmetry about X-axis $\Rightarrow y_0 = \frac{h}{2} = \frac{6\,cm}{2} = 3\,cm$

$$x_0 = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

→ A_1 = Area of whole section (assumed all solid)

$= h \times b = 6\,cm \times 10\,cm = 60\,cm^2$

→ x_1 = x coordinate of center of gravity of whole section

$= \frac{b}{2} = \frac{10\,cm}{2} = 5\,cm$

→ A_2 = Area of hollow portion $= (b - t_1)(h - 2t_2)$

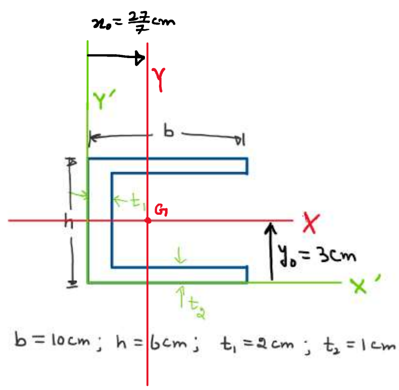
$= (10-2)(6-2\times1) = 8 \times 4 = 32\,cm^2$

→ x_2 = x coordinate of hollow portion $= t_1 + \frac{b - t_1}{2}$

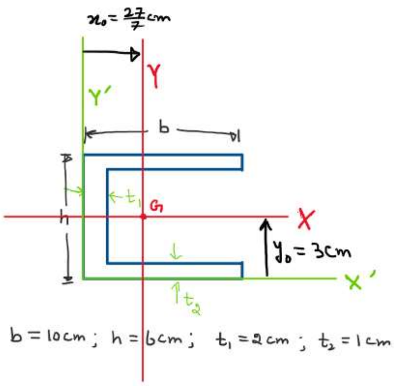
$= 2 + \frac{10-2}{2} = 2 + \frac{8}{2} = 2 + 4 = 6\,cm$

Plugging the values we get:

$$x_0 = \frac{27}{7}\,cm \simeq 3.86\,cm$$



Step-2: Find I_x and I_y:

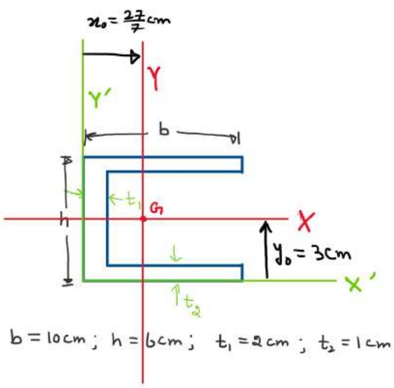


$$I_x = I_{1x} - I_{2x}$$

$$\rightarrow I_{1x} = \frac{bh^3}{12} = \frac{10 \times 6^3}{12} = 180 \text{ cm}^4$$

$$\rightarrow I_{2x} = \frac{(b-t_1)(h-2t_2)^3}{12} = \frac{(10-2)(6-2\times1)^3}{12} = \frac{128}{3} \simeq 42.67 \text{ cm}^4$$

$$\rightarrow I_x = 180 - \frac{128}{3} = \frac{412}{3} \text{ cm}^4 \simeq 137 \text{ cm}^4$$



$$I_y = I_{1y} - I_{2y}$$

$$\rightarrow I_{1y} = \frac{hb^3}{12} + \left[h \times b \times \left(\frac{b}{2} - x_0\right)^2\right] \text{ [Parallel axis Theorem]}$$

$$= \frac{6 \times 10^3}{12} + \left[6 \times 10 \times \left(\frac{10}{2} - \frac{27}{7}\right)^2\right] \simeq 578.37 \text{ cm}^4$$

$$\rightarrow I_{2y} = \frac{(h-2t_2)(b-t_1)^3}{12} + \left[(h-2t_2)(b-t_1)\left(t_1 + \frac{b-t_1}{2} - x_0\right)^2\right]$$

$$= \frac{(6-2\times1)(10-2)^3}{12} + \left[(6-2\times1)(10-2)\left(2 + \frac{10-2}{2} - \frac{27}{7}\right)^2\right]$$

$$= 317.6 \text{ cm}^4$$

$$\rightarrow I_y = 578.37 - 317.6 = 260.77 \text{ cm}^4$$