Solid Mechanics First and Second Moment of Area (T section) problem Solution using Method of Integration:

Find the Area, Centroid, Second Moment of Area, Polar Second moment of Area, Product moment of Area and Principal Axes of the T section given, using **integration method**

Areas

First moment of area

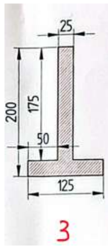
Centroid (find the x and y coordinates of the shape)

Second moment of area

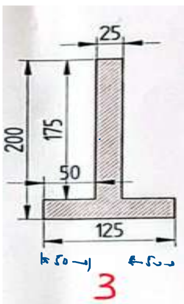
Polar second moment of area

Product moment of area

Principal axes



Area:

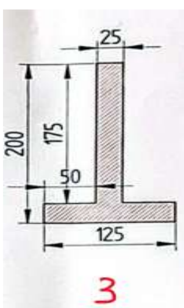


$$\int dA = \iint dx\,dy = \int_0^{125} dx \int_0^{200} dy - \int_0^{50} dx \int_{25}^{200} dy - \int_{75}^{125} dx \int_{25}^{200} dy$$

$$\int_0^{125} 1\,d[x] \times \int_0^{200} 1\,d[y] - \int_0^{50} 1\,d[x] \times \int_{25}^{200} 1\,d[y] - \int_{75}^{125} 1\,d[x] \times \int_{25}^{200} 1\,d[y]$$

$\boxed{7\,500}$ mm^2

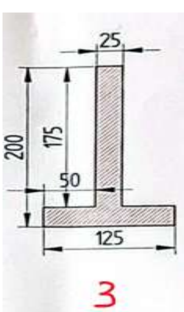
First Moment of Area(always w.r.t. Ref. axis):



$$S_x = \int y\,dA = \iint y\,dx\,dy = \int_0^{200} y\,dy \int_0^{125} dx - \int_{25}^{200} y\,dy \int_0^{50} dx - \int_{25}^{200} y\,dy \int_{75}^{125} dx$$

$$\int_0^{200} y\,d[y] \times \int_0^{125} 1\,d[x] - \int_{25}^{200} y\,d[y] \times \int_0^{50} 1\,d[x] - \int_{25}^{200} y\,d[y] \times \int_{75}^{125} 1\,d[x]$$

$S_x = \boxed{531\,250}$



$$S_y = \int x\,dA = \iint x\,dx\,dy = \int_0^{125} x\,dx \cdot \int_0^{200} dy - \int_0^{50} x\,dx \int_{25}^{200} dy - \int_{75}^{125} x\,dx \int_{25}^{200} dy$$

$S_y = \boxed{468\,750}$

Centroid:

Another method (not taught)

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(125\times 25\times 12.5) + (175\times 25\times(25+\frac{175}{2}))}{(125\times 25)+(175\times 25)} = 70.83$$

$S_y = \boxed{468\,750}$

$S_x = \boxed{531\,250}$

$\boxed{7\,500}$ mm^2

$$\bar{x} = \frac{S_y}{A} = \frac{468\,750}{7500} = 62.5 \text{ mm}$$

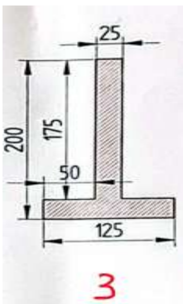
$$\bar{y} = \frac{S_x}{A} = \frac{531\,250}{A} = 70.8 \text{ mm}$$

$\bar{Y} = \frac{S_x}{A} = \frac{531\,250}{A} = 70.8$ mm

Second Moment of Area a.k.a.
Area moment of inertia
Using Parallel axis theorem:

$$I_{x'} = \int y^2 \, dA = \iint y^2 \, dx \, dy$$

$$= \int_0^{200} y^2 dy \cdot \int_0^{125} dx - \int_{25}^{200} y^2 dy \cdot \int_0^{50} dx - \int_{25}^{200} y^2 dy \int_{75}^{125} dx$$



$$\int_0^{200} y^2 \, d[y] \times \int_0^{125} 1 \, d[x] - \int_{25}^{200} y^2 \, d[y] \times \int_0^{50} 1 \, d[x] - \int_{25}^{200} y^2 \, d[y] \times \int_{75}^{125} 1 \, d[x]$$

67 187 500 = $I_{x'}$

$$I_{y'} = \int x^2 \, dA = \iint x^2 \, dx \, dy$$

$$= \int_0^{125} x^2 dx \int_0^{200} dy - \int_0^{50} x^2 dx \cdot \int_{25}^{200} dy - \int_{75}^{125} x^2 dx \cdot \int_{25}^{200} dy$$

Direct:

$$\int_0^{125} x^2 \, d[x] \times \int_0^{200} 1 \, d[y] - \int_0^{50} x^2 \, d[x] \times \int_{25}^{200} 1 \, d[y] - \int_{75}^{125} x^2 \, d[x] \times \int_{25}^{200} 1 \, d[y]$$

33 593 750 = $I_{y'}$

$\Delta y = \bar{y}$

$$I_{x'} = I_x + A \Delta y^2$$

$$I_x = I_{x'} - A \Delta y^2$$

$$= 29\,592\,700 \text{ mm}^4$$

$67\,187\,500 - (7\,500 \times 70.8^2)$

$I_x =$ 29 592 700

$$I_{y'} = I_y + A \Delta x^2$$

$$I_y = I_{y'} - A \Delta x^2$$

$$I_y = 4\,296\,875 \text{ mm}^4$$

$33\,593\,750 - (7\,500 \times 62.5^2)$

$I_y =$ 4 296 875

Polar second moment of area a.k.a.
Polar moment of inertia (about centroid)

$I_y =$ 4 296 875 mm^4

$I_x =$ 29 592 700 mm^4

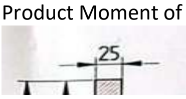
$4\,296\,875 + 29\,592\,700$

33 889 575

$$J = I_x + I_y$$

$$J = 33\,889\,575 \text{ mm}^4$$

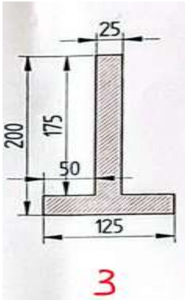
Product Moment of Area (about centroid)



$$I_{xy} = \int\int xy \, dA = \iint xy \, dx \, dy$$

$$= \int^{125} x \, dx \int^{200} y \, dy - \int^{50} x \, dx \int^{200} y \, dy - \int^{125} x \, dx \int^{200} y \, dy$$

Product Moment of Area (about centroid)



$$I_{xy} = \oint \int xy \, dA = \iint xy \, dx \, dy$$

$$= \int_0^{125} x\,dx \int_0^{200} y\,dy - \int_0^{50} x\,dx \int_{25}^{200} y\,dy - \int_{75}^{125} x\,dx \int_{25}^{200} y\,dy$$

$$= \frac{x^2}{2}\Big|_0^{125} \cdot \frac{y^2}{2}\Big|_0^{200} - \frac{x^2}{2}\Big|_0^{50} \cdot \frac{y^2}{2}\Big|_{25}^{200} - \frac{x^2}{2}\Big|_{75}^{125} \cdot \frac{y^2}{2}\Big|_{25}^{200}$$

$$\left(\frac{125^2}{2} \cdot \frac{200^2}{2}\right) - \left[\left(\frac{50^2}{2}\right) \cdot \left(\frac{200^2}{2} - \frac{25^2}{2}\right)\right] - \left[\left(\frac{125^2}{2} - \frac{75^2}{2}\right) \cdot \left(\frac{200^2}{2} - \frac{25^2}{2}\right)\right]$$

$$\int_0^{125} x\,d[x] \times \int_0^{200} y\,d[y] - \int_0^{50} x\,d[x] \times \int_{25}^{200} y\,d[y] - \int_{75}^{125} x\,d[x] \times \int_{25}^{200} y\,d[y]$$

$33\,203\,125 = I_{xy}$

Parallel axis Theorem:

$$I_{xy}' = I_{xy} + A\Delta x \Delta y$$

$$I_{xy} = I_{xy} - A\Delta x \Delta y \approx 0$$

$7\,500 \times 62.5 \times 70.833\,333\,333\,333\,33$

$0.000\,000\,001\,56 \approx 0$

Principal axes

Principal Angle:

$$\tan 2\alpha_p = \frac{-I_{xy}}{\frac{I_x - I_y}{2}}$$

$$\tan 2\alpha_p = 0$$

$$2\alpha_p = \tan^{-1} 0 = 0, 180$$

$$\alpha_p = 0, 90$$

Principal MOI:

$I_y = 4\,296\,875 \text{ mm}^4$

$I_x = 29\,592\,700 \text{ mm}^4$

$J = 33\,889\,575 \text{ mm}^4$

$$I = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

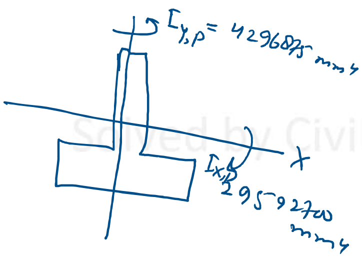
$I_{max} =$

$$\frac{33\,889\,575}{2} + \sqrt{\left(\frac{29\,592\,700 - 4\,296\,875}{2}\right)^2}$$

$= 29\,592\,700$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 0 - I_{xy} \sin 0$$

$29\,592\,700 \Rightarrow I_{x'}$



I_{min}

$$= \frac{33\,889\,575}{2} - \sqrt{\left(\frac{29\,592\,700 - 4\,296\,875}{2}\right)^2}$$

$= 4\,296\,875$