

problem Solution using **Method of Integration**:

Find the Area, Centroid, Second Moment of Area, Polar Second moment of Area, Product moment of Area and Principal Axes of the T section given, using **integration method**

Areas

First moment of area

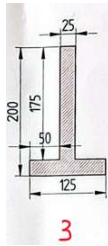
Centroid (find the x and y coordinates of the shape)

Second moment of area

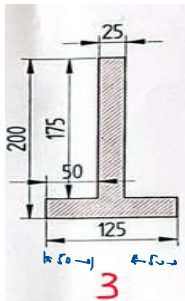
Polar second moment of area

Product moment of area

Principal axes



Area:

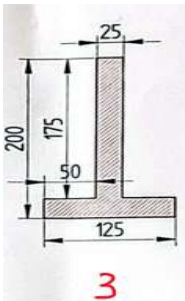


$$\int dA = \int_{-125}^{125} \int_0^{200} dx dy = \int_0^{200} \int_{-125}^{125} dy dx = \int_0^{200} [x]_{-125}^{125} dx = \int_0^{200} 250 dx = 250 \times 200 = 50000 \text{ mm}^2$$

$$\int_0^{200} \int_{-125}^{125} 1 d[x] \times 1 d[y] - \int_0^{200} \int_{-125}^{125} 1 d[x] \times \int_{-125}^{125} 1 d[y] = \int_0^{200} [x]_{-125}^{125} dx = \int_0^{200} 250 dx = 50000 \text{ mm}^2$$

$$7500 \text{ mm}^2$$

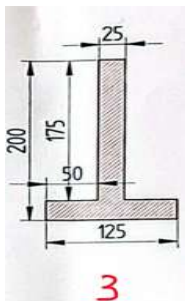
First Moment of Area(always w.r.t. Ref. axis):



$$S_x = \int y dA = \int_0^{200} \int_{-125}^{125} y dx dy = \int_0^{200} y [x]_{-125}^{125} dy = \int_0^{200} 250 y dy = 250 \times \frac{y^2}{2} \Big|_0^{200} = 250 \times \frac{200^2}{2} = 5000000 \text{ mm}^3$$

$$\int_0^{200} \int_{-125}^{125} y d[y] \times 1 d[x] - \int_0^{200} \int_{-125}^{125} y d[x] \times 1 d[y] = \int_0^{200} [y^2]_{-125}^{125} dy = \int_0^{200} 250 y dy = 5000000 \text{ mm}^3$$

$$S_x = 531250$$



$$S_y = \int x dA = \int_{-125}^{125} \int_0^{200} x dy dx = \int_{-125}^{125} x [y]_0^{200} dx = \int_{-125}^{125} 200 x dx = 200 \times \frac{x^2}{2} \Big|_{-125}^{125} = 200 \times \frac{125^2}{2} = 1562500 \text{ mm}^3$$

$$S_y = 468750$$

Centroid:

$$\bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2} = \frac{(125 \times 25) \times (25 \times \frac{125}{2}) + (175 \times 25) \times (25 \times \frac{175}{2})}{(125 \times 25) + (175 \times 25)} = 70.83$$

$$S_y = 468750$$

$$S_x = 531250$$

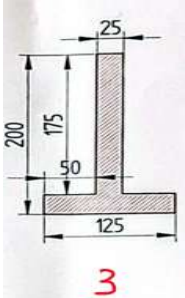
$$7500 \text{ mm}^2$$

$$\bar{x} = \frac{S_y}{A} = \frac{468750}{7500} = 62.5 \text{ mm}$$

$$\bar{y} = \frac{S_x}{A} = \frac{531250}{7500} = 70.8 \text{ mm}$$

$$\bar{y} = \frac{Sx}{A} = \frac{531250}{A} = 70.8 \text{ mm}$$

Second Moment of Area a.k.a.
Area moment of inertia
Using Parallel axis theorem:



Direct:

Solved by Civil Thinking (<https://civilthinking.com>)

$$\begin{aligned} I_{x'} &= \int y^2 dA = \iint y^2 dx dy \\ &= \int_0^{200} y^2 dy \cdot \int_0^{125} dx - \int_{25}^{200} y^2 dy \cdot \int_0^{125} dx \\ &\quad - \int_{25}^{125} y^2 dy \cdot \int_{75}^{125} dx \end{aligned}$$

$$\int_0^{200} y^2 d[y] \times \int_0^{125} 1 d[x] - \int_{25}^{200} y^2 d[y] \times \int_0^{125} 1 d[x] - \int_{75}^{125} y^2 d[y] \times \int_{75}^{125} 1 d[x]$$

$$67\ 187\ 500 \text{ } I_{x'}$$

$$\begin{aligned} I_{y'} &= \int x^2 dA = \iint x^2 dx dy \\ &= \int_0^{125} x^2 dx \cdot \int_0^{200} dy - \int_0^{125} x^2 dx \cdot \int_{25}^{200} dy \\ &\quad - \int_{75}^{125} x^2 dx \cdot \int_{75}^{125} dy \end{aligned}$$

$$\int_0^{125} x^2 d[x] \times \int_0^{200} 1 d[y] - \int_0^{125} x^2 d[x] \times \int_{25}^{200} 1 d[y] - \int_{75}^{125} x^2 d[x] \times \int_{75}^{125} 1 d[y]$$

$$33\ 593\ 750 \text{ } I_{y'}$$

$$\begin{aligned} I_{x'} &= I_x + A \Delta y^2 \\ I_x &= I_{x'} - A \Delta y^2 \\ &= 29\ 592\ 700 \text{ mm}^4 \end{aligned}$$

$$67\ 187\ 500 - (7\ 500 \times 70.8^2)$$

$$I_x = 29\ 592\ 700$$

$$\begin{aligned} I_{y'} &= I_y + A \Delta x^2 \\ I_y &= I_{y'} - A \Delta x^2 \\ I_y &= 4\ 296\ 875 \text{ mm}^4 \end{aligned}$$

$$33\ 593\ 750 - (7\ 500 \times 62.5^2)$$

$$I_y = 4\ 296\ 875$$

Polar second moment of area a.k.a.
Polar moment of inertia (about centroid)

$$I_y = 4\ 296\ 875 \text{ mm}^4$$

$$I_x = 29\ 592\ 700 \text{ mm}^4$$

$$4\ 296\ 875 + 29\ 592\ 700$$

$$33\ 889\ 575$$

$$J = I_x + I_y$$

$$J = 33\ 889\ 575 \text{ mm}^4$$

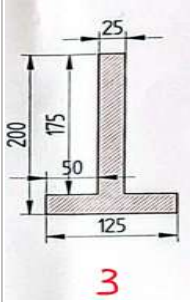
Product Moment of Area (about centroid)



$$I_{xy} = \int xy dA = \iint xy dx dy$$

$$= \int_0^{125} x dx \int_0^{200} y dy - \int_0^{125} x dx \int_{25}^{200} y dy - \int_{75}^{125} x dx \int_{75}^{125} y dy$$

Product Moment of Area (about centroid)



$$\begin{aligned}
 I_{xy} &= \int_0^{125} \int_0^{200} xy \, dA = \int_0^{125} \int_0^{200} xy \, dx \, dy \\
 &= \int_0^{125} x \, dx \int_0^{200} y \, dy - \int_0^{50} x \, dx \int_0^{200} y \, dy - \int_{50}^{125} x \, dx \int_0^{50} y \, dy \\
 &= \left[\frac{x^2}{2} \right]_0^{125} \cdot \left[\frac{y^2}{2} \right]_0^{200} - \left[\frac{x^2}{2} \right]_0^{50} \cdot \left[\frac{y^2}{2} \right]_0^{200} - \left[\frac{x^2}{2} \right]_{50}^{125} \cdot \left[\frac{y^2}{2} \right]_0^{50} \\
 &= \left(\frac{125^2}{2} - \frac{200^2}{2} \right) - \left[\left(\frac{50^2}{2} \right) \cdot \left(\frac{200^2}{2} - \frac{25^2}{2} \right) \right] - \left[\left(\frac{125^2}{2} - \frac{75^2}{2} \right) \cdot \left(\frac{200^2}{2} - \frac{25^2}{2} \right) \right]
 \end{aligned}$$

$$\int_0^{125} x \, dx \int_0^{200} y \, dy - \int_0^{50} x \, dx \int_0^{200} y \, dy - \int_{50}^{125} x \, dx \int_0^{50} y \, dy$$

$$33 \, 203 \, 125 = I_{xy}$$

Parallel axis Theorem:

$$\begin{aligned}
 I_{xy}' &= I_{xy} + A \Delta x \Delta y \\
 I_{xy} &= I_{xy} - A \Delta x \Delta y = 0
 \end{aligned}$$

$$\begin{aligned}
 &7 \, 500 \times 62.5 \times 70.833 \, 333 \, 333 \, 33 \, 3 \\
 &0.000 \, 000 \, 001 \, 56 \, 0
 \end{aligned}$$

Principal axes

Principal Angle:

$$\tan 2\alpha_p = \frac{-I_{xy}}{\frac{I_x - I_y}{2}}$$

$$\begin{aligned}
 \tan 2\alpha_p &= 0 \\
 2\alpha_p &= \tan^{-1} 0 = 0, 180^\circ \\
 \alpha_p &= 0, 90^\circ
 \end{aligned}$$

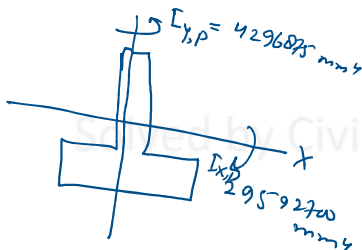
Principal MOI:

$$\begin{aligned}
 I_y &= 4 \, 296 \, 875 \, \text{mm}^4 \\
 I_x &= 29 \, 592 \, 700 \, \text{mm}^4
 \end{aligned}$$

$$J = 33 \, 889 \, 575 \, \text{mm}^4$$

$$I_x' = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 0 - I_{xy} \sin 0$$

$$29 \, 592 \, 700 \Rightarrow I_{x'}$$



$$\begin{aligned}
 &\frac{33 \, 889 \, 575}{2} + \sqrt{\left(\frac{29 \, 592 \, 700 - 4 \, 296 \, 875}{2} \right)^2} \\
 &= 29 \, 592 \, 700
 \end{aligned}$$

$$I_{min}$$

$$\begin{aligned}
 &= \frac{33 \, 889 \, 575}{2} - \sqrt{\left(\frac{29 \, 592 \, 700 - 4 \, 296 \, 875}{2} \right)^2} \\
 &= 4 \, 296 \, 875
 \end{aligned}$$

This problem was solved by Civil Thinking (<https://civilthinking.com>)

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