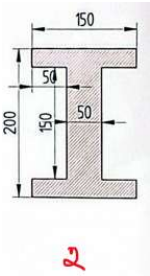
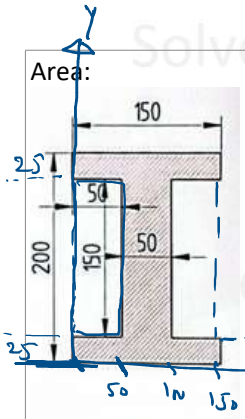
Solid Mechanics First and Second Moment of Area
problem Solution:

Find the Area, Centroid, Second Moment of Area, Polar Second moment of Area, Product moment of Area and Principal Axes of the I section given, using integration method



Area:

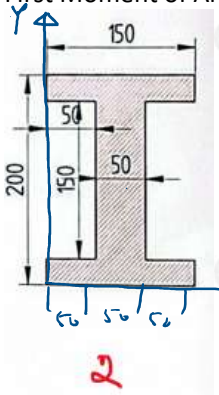


$$A = \int dA = \iint dx\,dy - \iint dx\,dy - \iint dx\,dy$$

$$= \int_0^{150}\int_0^{200} dx\,dy - \int_0^{50}\int_{25}^{175} dx\,dy - \int_{100}^{150}\int_{25}^{175} dx\,dy$$

$$= 15000\ mm^2$$

First Moment of Area(Always i.t.o. Reference):



$$S_x = \int y\,dA = \iint y\,dx\,dy = \int_0^{200}\int_{150}^{175} y\,dx\,dy - \int_0^{50}\int_{175}^{150} y\,dx\,dy - \int_{25}^{100}\int_{150}^{150} y\,dx\,dy$$

$$= \int_0^{200} y\,dy \int_0^{25} dx - \int_0^{50} y\,dy \int_0^{25} dx - \int_{25}^{100} y\,dy \int_0^{25} dx = 15 \times 10^5\ mm^3$$

$$\int_0^{200} y\,d[y] \times \int_0^{150} 1\,d[x] - \int_0^{50} y\,d[y] \times \int_0^{25} 1\,d[x] - \int_{25}^{100} y\,d[y] \times \int_0^{25} 1\,d[x]$$

1 500 000

$$S_y = \int x\,dA = \iint x\,dx\,dy = \int_0^{150}\int_0^{200} x\,dx\,dy - \int_0^{50}\int_{25}^{175} x\,dx\,dy - \int_{100}^{150}\int_{25}^{175} x\,dx\,dy$$

$$= 1125000 = 11.25 \times 10^5\ mm^3$$

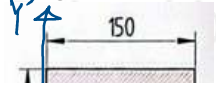
$$\int_0^{150} x\,d[x] \times \int_0^{200} 1\,d[y] - \int_0^{50} x\,d[x] \times \int_{25}^{175} 1\,d[y] - \int_{100}^{150} x\,d[x] \times \int_{25}^{175} 1\,d[y]$$

1 125 000

Centroid:

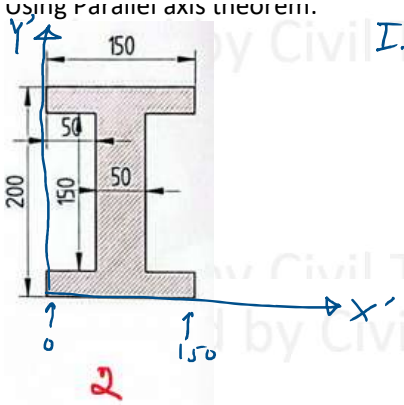
$$\bar{X} = \frac{S_y}{A} = \frac{11.25 \times 10^5}{15000} = 75\ mm\ ;\ \bar{Y} = \frac{S_x}{A} = 100$$

Second Moment of Area a.k.a.
Area moment of inertia
Using Parallel axis theorem:



$$I_{x'} = \int y^2\,dA = \iint y^2\,dx\,dy = \int_0^{200} y^2\,dy \int_0^{150} dx - \int_{25}^{175} y^2\,dy \int_0^{50} dx$$

Using Parallel axis theorem:



$$I_{x'} = \int y^2 dA = \iint y^2 dx dy = \int_0^{200} y^2 dy \int_0^{150} dx - \int_{25}^{175} y^2 dy \int_0^{50} dx - \int_{25}^{175} y^2 dy \int_{100}^{150} dx$$

$= 221\,875\,000$

$$\int_0^{200} y^2 d[y] \times \int_0^{150} 1 d[x] - \int_{25}^{175} y^2 d[y] \times \int_0^{50} 1 d[x] - \int_{25}^{175} y^2 d[y] \times \int_{100}^{150} 1 d[x]$$

221 875 000

$$I_{y'} = \int x^2 dA = \iint x^2 dx dy = \int_0^{150} x^2 dx \int_0^{200} dy - \int_0^{150} x^2 dx \int_{25}^{175} dy - \int_{100}^{150} x^2 dx \int_{25}^{175} dy$$

$= 100\,000\,000 \text{ mm}^4$

$$\int_0^{150} x^2 d[x] \times \int_0^{200} 1 d[y] - \int_0^{50} x^2 d[x] \times \int_{25}^{175} 1 d[y] - \int_{100}^{150} x^2 d[x] \times \int_{25}^{175} 1 d[y]$$

100 000 000

PAT:

$I_{x'} = I_x + A \Delta y^2$

$\Rightarrow I_x = I_{x'} - A \Delta y^2$

$I_{y'} = I_y + A \Delta x^2$

$I_y = I_{y'} - A \Delta x^2 = 15625000 \text{ mm}^4$

221875000-15000*100^2=71875000 mm^4

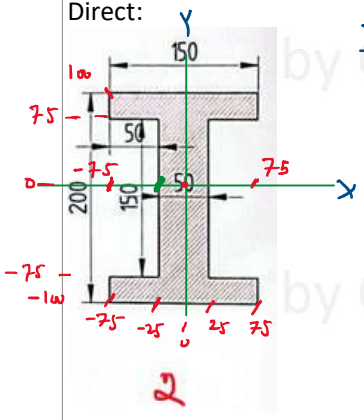
221 875 000 - (15 000 × 100²)

71 875 000

100 000 000 - (15 000 × 75²)

15 625 000

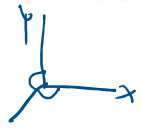
Direct:



$$I_x = \iint y^2 dx dy = \int_{-100}^{100} y^2 dy \int_{-75}^{75} dx - \int_{-75}^{75} y^2 dy \int_{-75}^{-25} dx - \int_{-75}^{75} y^2 dy \int_{25}^{75} dx$$

$$\int_{-100}^{100} y^2 d[y] \times \int_{-75}^{75} 1 d[x] - \int_{-75}^{75} y^2 d[y] \times \int_{-75}^{-25} 1 d[x] - \int_{-75}^{75} y^2 d[y] \times \int_{25}^{75} 1 d[x]$$

71 875 000

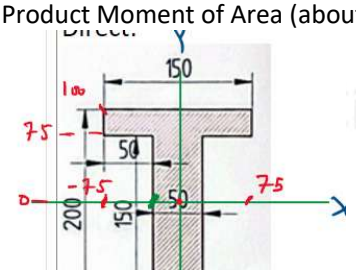


Polar second moment of area a.k.a.
Polar moment of inertia (about centroid)

$J = I_x + I_y$ (Theorem of Perpendicular Axis)

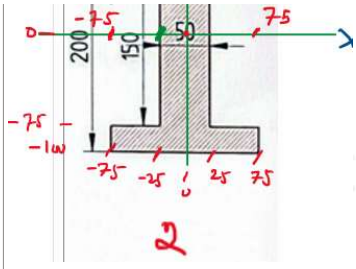
$71875000 + 15625000 = 87500000 \text{ mm}^4$

Product Moment of Area (about centroid):



$$I_{xy} = \iint xy dx dy = \int_{-25}^{75} x dx \int_{-75}^{100} y dy - \int_{75}^{75} x dx \int_{75}^{-75} y dy - \int_{25}^{75} x dx \int_{-75}^{-75} y dy = 0$$

0



0

Principal axes:

Principal Angle: $\tan 2\alpha_p = -\frac{I_{xy}}{\frac{I_x - I_y}{2}} = \frac{\text{Product}}{\text{Avg. Diff.}} = 0$

$\tan 2\alpha_p = 0 \Rightarrow 2\alpha_p = \tan^{-1} 0 = 0°, 180°$

$\Rightarrow 2\alpha_p = 0°, 180° \Rightarrow \alpha_p = 0°, 90°$

Principal MOI:

$$I = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$\frac{71875000 + 15625000}{2} \pm \sqrt{\left(\frac{71875000 - 15625000}{2}\right)^2 + 0^2}$$

$I_{max} = 71875000\ mm^4$

$$\frac{87\,500\,000}{2} + \sqrt{\left(\frac{71\,875\,000 - 15\,625\,000}{2}\right)^2 + 0^2}$$

71 875 000

$I_{min} = 15625000$

$$\frac{87\,500\,000}{2} - \sqrt{\left(\frac{71\,875\,000 - 15\,625\,000}{2}\right)^2 + 0^2}$$

15 625 000

Cross check Principal Moment of inertia:

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\alpha - I_{xy}\sin 2\alpha$$

$\alpha \rightarrow \alpha_p \rightarrow I_{max/min}$

$$\frac{87\,500\,000}{2} + \frac{71\,875\,000 - 15\,625\,000}{2} \times \text{Cos}(2 \times 0) - 0$$

71 875 000

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