Hydrodynamics problem solved using Modified Bernoulli's Equation

Oil with a viscosity of $\upsilon = 0{,}6 \cdot 10^{-4} \frac{m^2}{s}$ is delivered from a pump to a lubricated sliding surface through a tube with a diameter $d = 0{,}01m$ and a length $l = 6m$. For what pressure difference will a volume flow rat $Q = 50 \frac{cm^3}{s}$ be provided? The density of the oil is $\rho = 890 \frac{kg}{m^3}$.

$\rho g = \gamma$

Solution:

The hydraulic head loss is defined as the ratio of the pressure drop to the specific weight of the fluid (Darcy's formula):

$$h_{st} = \frac{\Delta p}{\gamma} = \lambda \frac{l}{d} \frac{u^2}{2g} \qquad (11.1)$$

$$\Rightarrow \Delta P = \lambda \frac{L}{d} \frac{u^2}{2g} \gamma$$

$$\lambda = f(Re)$$

$$Re = \frac{VD}{\nu} \; ; \; u = \frac{Q}{A} = \frac{50 \times 10^{-6} \, m^3/s}{\frac{\pi}{4} \, 0.01^2 \, m^2} = 0.6366 \, m/s$$

$$Re = \frac{0.6366 \times 0.01 \, m}{0.6 \times 10^{-4} \, m^2/s} = 106.1 < 2320$$

$\Rightarrow$ laminar flow

$$\Rightarrow \lambda = \frac{64}{Re} = \frac{64}{106.1} = 0.6032$$

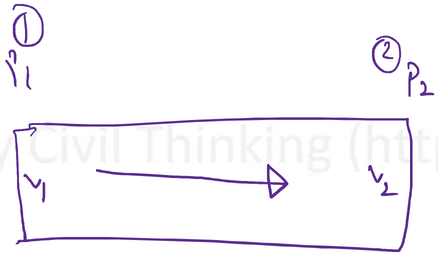
$$\Delta P = \lambda \frac{L}{d} \frac{u^2}{2g} \gamma$$

$\lambda = 0.6032$
$L = 6 \, m$
$d = 0.01 \, m$
$u = 0.6366 \, m/s$
$\gamma = \rho g = 890 \times 9.81$

$$\Rightarrow \Delta P = 65268.83 \, Pa$$
$$\simeq 65.3 \, kPa$$

ANS.

We can also solve this using modified Bernoulli eqn & continuity eqn



$h_L = h_f + h_l$

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 - h_L = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2} \longrightarrow \text{Eqn ①}$$

→ $z_1 = z_2$ [Horizontal pipe]

→ $v_1, v_2 = ?$

continuity eqn

$\dot{m}_1 = \dot{m}_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

same fluid ⇒ $\rho_1 = \rho_2$

→ $A_1 v_1 = A_2 v_2$

same cross s/c^n ⇒ $A_1 = A_2$

→ $v_1 = v_2 = \dfrac{Q}{A}$

$$= \frac{50 \times 10^{-6}\ m^3/s}{\frac{\pi}{4}\ 0.01^2\ m^2} = 0.6366\ m/s$$

$v_1 = v_2 = 0.6366\ m/s$

→ $p_1, p_2 = ?$ To find $p_1 - p_2 = \Delta p$

→ $h_L = ?$

$$h_L = \frac{\lambda l v^2}{2gd}$$

$$\lambda = f(Re)$$

$Re = \dfrac{VD}{\nu}$; $u = \dfrac{Q}{A} = \dfrac{50 \times 10^{-6}\ m^3/s}{\frac{\pi}{4}\ 0.01^2\ m^2} = 0.6366\ m/s$

$$Re = \frac{VD}{\nu} \quad ; \quad u = \frac{Q}{A} = \frac{50\times10^{-6}\, m^3/s}{\frac{\pi}{4}\, 0.01^2\, m^2} = 0.6366\, m/s$$

$$Re = \frac{0.6366 \quad \times 0.01\, m}{0.6\times10^{-4}\, m^2/s} = 106.1 < 2320$$

$\Rightarrow$ laminar flow

$$\Rightarrow \lambda = \frac{64}{Re} = \frac{64}{106.1} = 0.6032$$

$$h_L = \frac{0.6032 \times 6\, m \times 0.6366^2}{2\times 9.81 \times 0.01} = 7.4756\ m$$

$$\boxed{\frac{P_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} + \cancel{z_1} - h_L = \frac{P_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + \cancel{z_2}}$$

Eqn 1 (copied again for reference)

Eqn 1 simplifies to:

$$\frac{P_1 - P_2}{\rho g} - h_L = \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{\Delta P}{\rho g} - h_L = \frac{V_2^2}{2g}$$

$$\Rightarrow \Delta P = \left(\frac{V_2^2}{2g} + h_L\right)\times \rho g$$

$$\Rightarrow \Delta P = \left(\frac{0.6366^2}{2g} + 7.4756\ m\right) \times 890 \times 9.81$$

$$= 65449\ Pa \simeq 65.4\ KPa \quad \underline{\underline{Ans}}$$

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