

Hydrodynamics problem solved using Modified Bernoulli's Equation

Oil with a viscosity of $\nu = 0,6 \cdot 10^{-4} \frac{m^2}{s}$ is delivered from a pump to a lubricated sliding surface through a tube with a diameter $d = 0,01m$ and a length $l = 6m$. For what pressure difference will a volume flow rate $Q = 50 \frac{cm^3}{s}$ be provided?

The density of the oil is $\rho = 890 \frac{kg}{m^3}$.

$$\rho g = \gamma$$

Solution:

The hydraulic head loss is defined as the ratio of the pressure drop to the specific weight of the fluid (Darcy's formula):

$$h_{st} = \frac{\Delta p}{\gamma} = \lambda \frac{l}{d} \frac{u^2}{2g} \quad (11.1)$$

$$\Rightarrow \Delta p = \lambda \frac{l}{d} \frac{u^2}{2g} \gamma$$

$$\lambda = f(Re)$$

$$Re = \frac{VD}{\nu} \quad ; \quad u = \frac{Q}{A} = \frac{50 \times 10^{-6} m^3/s}{\frac{\pi}{4} 0.01^2 m^2} = 0.6366 m/s$$

$$Re = \frac{0.6366 \times 0.01 m}{0.6 \times 10^{-4} \frac{m^2}{s}} = 106.1 < 2320 \Rightarrow \text{laminar flow}$$

$$\Rightarrow \lambda = \frac{64}{Re} = \frac{64}{106.1} = 0.6032$$

$$\Delta p = \lambda \frac{l}{d} \frac{u^2}{2g} \gamma$$

$$\Rightarrow \Delta p = 65268.83 Pa \\ \approx 65.3 kPa \\ \underline{\underline{Ans.}}$$

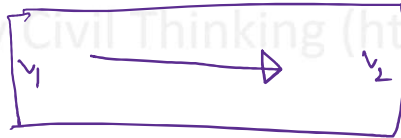
$$\left\{ \begin{array}{l} \lambda = 0.6032 \\ L = 6 m \\ d = 0.01 m \\ u = 0.6366 m/s \\ \gamma = \rho g = 890 \times 9.81 \end{array} \right.$$

We can also solve this using modified Bernoulli eqn & continuity eqn

①
 P_1

②
 P_2

continuity eqn



$$h_L = h_f + h_L$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 - h_L = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 \quad \text{Eqn (1)}$$

$$\rightarrow Z_1 = Z_2 \quad [\text{Horizontal Pipe}]$$

$$\rightarrow v_1, v_2 = ?$$

continuity eqn

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$\text{same fluid} \Rightarrow \rho_1 = \rho_2$$

$$A_1 v_1 = A_2 v_2$$

$$\text{same cross section} \Rightarrow A_1 = A_2$$

$$v_1 = v_2 = \frac{Q}{A}$$

$$= \frac{50 \times 10^{-6} \text{ m}^3/\text{s}}{\frac{\pi}{4} 0.01^2 \text{ m}^2} = 0.6366 \text{ m/s}$$

$$v_1 = v_2 = 0.6366 \text{ m/s}$$

$$\rightarrow P_1, P_2 = ? \quad \text{To find } P_1 - P_2 = \Delta P$$

$$\rightarrow h_L = ?$$

$$h_L = \frac{\lambda l v^2}{2g d}$$

$$\lambda = f(Re)$$

$$Re = \frac{VD}{\nu} \quad ; \quad \nu = \frac{Q}{A} = \frac{50 \times 10^{-6} \text{ m}^3/\text{s}}{\frac{\pi}{4} 0.01^2 \text{ m}^2} = 0.6366 \text{ m/s}$$

$$Re = \frac{VD}{\nu} \quad ; \quad \nu = \frac{Q}{A} = \frac{50 \times 10^{-6} \text{ m}^3/\text{s}}{\frac{\pi}{4} (0.01)^2 \text{ m}^2} = 0.6366 \text{ m/s}$$

$$Re = \frac{0.6366 \times 0.01 \text{ m}}{0.6 \times 10^{-4} \text{ m}^2/\text{s}} = 106.1 < 2320 \Rightarrow \text{laminar flow}$$

$$\Rightarrow \lambda = \frac{64}{Re} = \frac{64}{106.1} = 0.6032$$

$$h_L = \frac{0.6032 \times 6 \text{ m} \times 0.6366^2}{2 \times 9.81 \times 0.01} = 7.4756 \text{ m}$$

$$\boxed{\frac{P_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} + \cancel{z_1} - h_L = \frac{P_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + \cancel{z_2}} \quad \text{Eqn (1)}$$

(copied again for reference)

Eqn (1) simplifies to:

$$\frac{P_1 - P_2}{\rho g} - h_L = \frac{v_2^2}{2g}$$

$$\Rightarrow \frac{\Delta P}{\rho g} - h_L = \frac{v_2^2}{2g}$$

$$\Rightarrow \Delta P = \left(\frac{v_2^2}{2g} + h_L \right) \times \rho g$$

$$\Rightarrow \Delta P = \left(\frac{0.6366^2}{2g} + 7.4756 \text{ m} \right) \times 890 \times 9.81$$

$$= 65449 \text{ Pa} = 65.4 \text{ kPa} \quad \underline{\underline{\text{Ans}}}$$

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