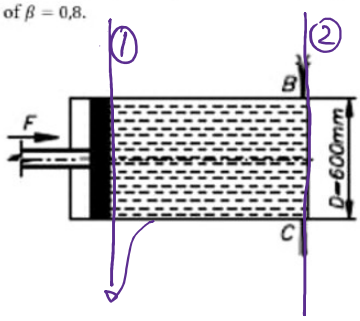
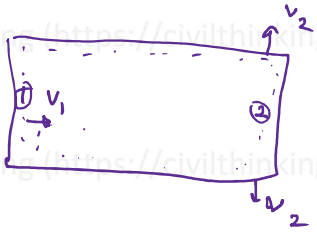
At what velocity does water flow out of the depicted cylinder through the openings B and C if the force acting on the piston is $F = 1500N$? How far does the piston move within a minute if the cross-sectional area of each opening is $A = 3mm^2$? Assume a contraction coefficient of $\beta = 0{,}8$.





$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$v_1 \approx 0$, $z_1 = z_2$, $p_2 = p_{atm} = 0$ [Gauge pressure]

$$\frac{p_1}{\rho g} = \frac{v_2^2}{2g} \Rightarrow v_2^2 = \frac{2p_1}{\rho}$$

$$\Rightarrow v_2 = \sqrt{\frac{2p_1}{\rho}} \leftarrow \text{we need } p_1 \text{ \& } \rho$$

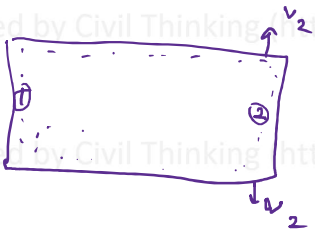
$$p_1 = \frac{F}{A} = \frac{1500\,N}{\frac{\pi}{4} \times 0.6^2} = 5305.16477\ N/m^2 \text{ (or Pa, pascal)}$$

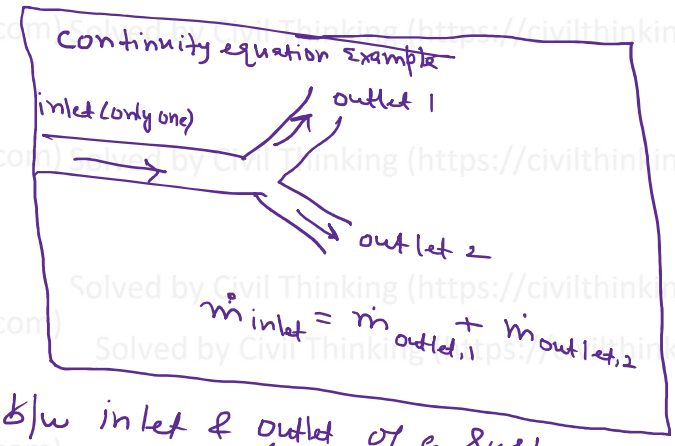
$$\Rightarrow v_2 = \sqrt{\frac{2 \times 5305.16477}{1000\,kg/m^3}} \approx 3.26\ m/s \quad \text{[Theoretical]}$$

$$v_{2,\,actual} = \beta \times v_2 = 0.8 \times 3.26 = 2.608\ m/s \quad \underline{\text{ANS. 1}}$$

Distance travelled by piston in a minute (60 s):

continuity equation example





continuity equation b/w ① & ②

↳ equal mass flow rate (kg/s) b/w inlet & outlet (exit) of a system

$$\sum \dot{m}_{inlet} = \sum \dot{m}_{outlet}$$

we have one inlet & two outlets

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 + \rho_2 A_2 V_2 \quad [\because \text{Velocity \& Area of both outlets is same, } V_2, A_2]$$

$\rho_1 = \rho_2$ [incompressible fluid]

$$\Rightarrow A_1 V_1 = A_2 V_2 + A_2 V_2 = 2 A_2 V_2$$

$$\Rightarrow \boxed{A_1 V_1 = 2 A_2 V_2}$$

$A_2 = 3\ (mm)^2 = 3 \times 10^{-6}\ m^2$ [Given] $\quad \left[3 \times (10^{-3}\ m)^2 = 3 \times 10^{-6}\ m \right]$

$V_2 = 2.608\ m/s$

$$\Rightarrow \frac{\pi}{4} 0.6^2 \times V_1 = 2 \times 3 \times 10^{-6} \times 2.608 =$$

$$\Rightarrow V_1 \simeq 0.55 \times 10^{-4}\ m/s$$

$$\text{Velocity} = \frac{\text{Distance}}{\text{time}}$$

$\Rightarrow$ Distance travelled = Velocity × time

$= 0.55 \times 10^{-4}\ m/s \times 1.0\ s$

$\Rightarrow$ Distance travelled = velocity x time

$= 0.55 \times 10^{-4}\ m/s \times 60\ s$

$\simeq 3.32 \times 10^{-3}\ m = 3.32\ mm$

Ans. 2