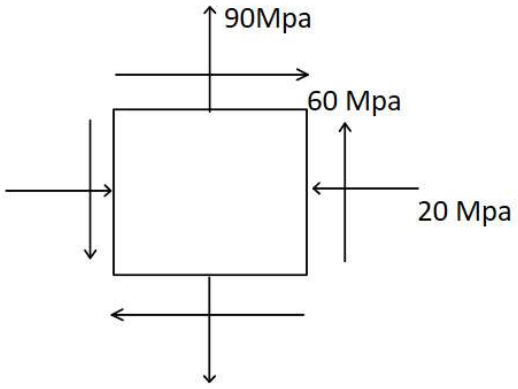
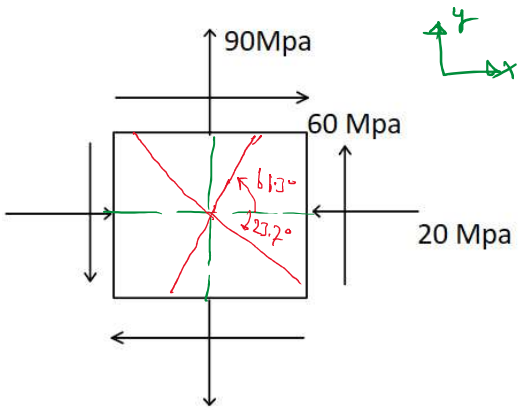
In-plane Principal Stresses : Question Solving Tutorial

Find the location and magnitude of in-plane major and minor principal stresses:



Solution:



Location:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$\sigma_x = -20$ mpa; $\sigma_y = 90$ mpa; $\tau_{xy} = 60$ mpa

$$\Rightarrow \tan 2\theta_p = \frac{60}{\frac{-20-90}{2}} = -23.7^\circ, 66.3^\circ$$

$$\sigma_y = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

($\sigma_y \downarrow \sigma_p$; $\theta \downarrow \theta_p$)

At $\theta_p = -23.7^\circ$

$$= \frac{-20+90}{2} + \frac{-20-90}{2}\cos 2(-23.7) + 60\sin 2(-23.7) = -46.4 \text{ MPa}$$

At $\theta_p = 66.3^\circ$

$= 116.4$ MPa

$\Rightarrow \sigma_{p_1} = 116.4$ mpa @ $\theta_{p_1} = 66.3^\circ$; $\sigma_{p_2} = -46.4$ mpa @ $\theta_{p_2} = -23.7^\circ$