Torsion in Shafts. Question 5-39 Solution

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5-39. The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.

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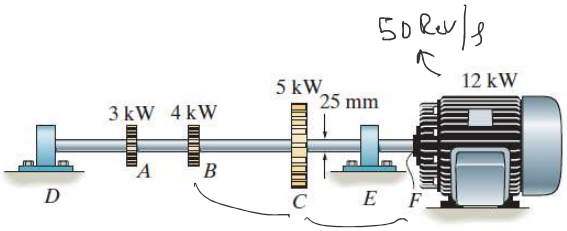
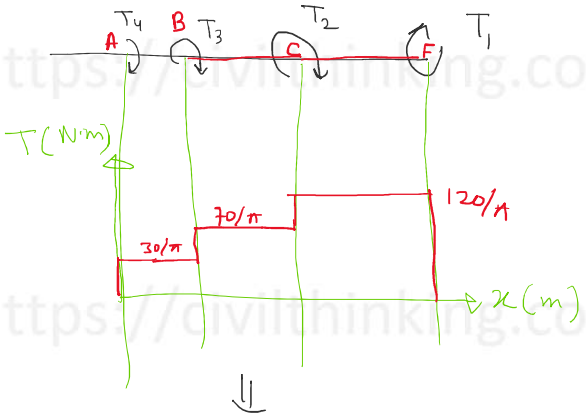


$$T_1 = \frac{P}{\omega} = \frac{12\times10^3 \text{ watt}}{2\pi\times50} = 120/\pi$$

$$T_2 = \frac{5\times10^3}{2\pi\times50} = 50/\pi$$

$$T_3 = \frac{4\times10^3}{2\pi\times50} = 40/\pi$$

$$T_4 = \frac{3\times10^3}{2\pi\times50} = 30/\pi$$



$T_{max,BC} = 70/\pi$ N.m ; $T_{max,CF} = 120/\pi$

To find: $\tau_{max,BC}$, $\tau_{max,CF}$

Let's apply Torsion equations to BC & CF:

$$\frac{\tau}{R} = \frac{T}{J}$$

In BC:

$$\frac{\tau_{BC}}{R_{BC}} = \frac{T}{J_{BC}} \Rightarrow \frac{\tau_{BC}}{\frac{0.0250}{2}} = \frac{70/\pi}{\frac{\pi}{2}\left(\frac{0.025}{2}\right)^4}$$

$$\left[\because R = \frac{\text{diameter}}{2} = \frac{25\text{mm}}{2} = \frac{0.0250}{2}\text{ m}\right]$$

$$\Rightarrow \tau_{BC} = \frac{7168000}{\pi^2} \text{ N/m}^2 = 7.26 \text{ MPa}$$

In CF:

$$\frac{\tau_{cf}}{\frac{0.0250}{2}} = \frac{120/\pi}{\frac{\pi}{2}\left(\frac{0.025}{2}\right)^4} \Rightarrow \tau_{cf} = \frac{122\,880\,000}{\pi^2}\ N/m^2 \approx 12.45\ MPa$$